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# Dynamic Loading of Spur Gears With Linear or Parabolic Tooth Profile Modifications





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# DYNAMIC LOADING OF SPUR GEARS WITH LINEAR OR PARABOLIC TOOTH PROFILE MODIFICATIONS

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## ABSTRACT

A computer simulation was conducted to investigate the effects of both linear and parabolic tooth profile modification on the dynamic response of low-contact-ratio spur gears. The effect of the total amount of modification and the length of the modification zone were studied at various loads and speeds to find the optimal profile modification for minimal dynamic loading.

Design charts consisting of normalized maximum dynamic load curves were generated for gear systems operated at various loads and with different tooth profile modification. An optimum profile modification can be determined from these design charts to minimize the dynamic loads of spur gear systems.

## NOMENCLATURE

$C_g$ damping coefficient of gear tooth mesh, N-sec (lb-sec)

$C_s$ damping coefficient of shaft, N-m-sec (in.-lb-sec)

$J_L$ polar moment of inertia of load, kg-m$^2$ (in.-lb-sec$^2$)

$J_M$ polar moment of inertia of motor, kg-m$^2$ (in.-lb-sec$^2$)

$J_1$ polar moment of inertia of gear 1, kg-m$^2$ (in.-lb-sec$^2$)

$J_2$ polar moment of inertia of gear 2, kg-m$^2$ (in.lb-sec$^2$)

$K_d$ dynamic factor

$K_g$ stiffness of gear tooth, N/m (lb/in.)

$K_s$ stiffness of shaft, N-m/rad (in.-lb/rad)

$L_n$ normalized length of tooth profile modification zone defined such that $L_n = 1.0$ is the length from tooth tip to HPSTC, measured along the line of contact.

$R_b$ base radius, mm (in.)

$T_L$ torque on load, N-m (in.-lb)

$T_M$ torque on motor, N-m (in.-lb)

$T_{f1}$ torque on gear 1, N-m (in.-lb)

$T_{f2}$ torque on gear 2, N-m (in.-lb)

$W_n$ normalized total transmitted load

$\theta$ angular displacement, rad

$\dot{\theta}$ angular velocity, rad/sec

$\ddot{\theta}$ angular acceleration, rad/sec$^2$

$\Delta$ amount of profile modification (thickness of material removed from tip of involute gear tooth), defined such that $\Delta = 1.0$ is the minimum amount of tip relief recommended by Welbourn, μm

## INTRODUCTION

One of the major concerns in the design of power transmission gears is the reduction of gear dynamic load. Research on gear noise and vibration has revealed that the basic mechanism of noise generated from gearing is gear box vibration excited by the dynamic load. Vibration is transmitted through shafts and bearings to noise-radiating surfaces on the exterior of the gear box. Dynamic load creates cyclic bending stresses in tooth roots which can lead to fatigue failure as well as cyclic subsurface stresses which can cause tooth surface failure by pitting and scoring. The life and reliability of a gear transmission is reduced by high dynamic load. Minimizing gear

dynamic load will decrease gear noise, increase efficiency, improve pitting fatigue life, and help prevent gear tooth fracture (1-5).

Modifying gear tooth profile is a widely used practice to reduce dynamic load for improved performance of a spur gear transmission. Current practice in gear design is to modify the tooth profile based on the maximum applied torque, also called design torque. When a modified gear system is operated at off-design torque, its dynamic load may become significant.

Research efforts have been conducted in this area for many years, yet there is a lack of systematic work leading to in-depth understanding of how tooth profile modifications affect the dynamic response of a spur gear transmission (1,4,6-9).

If the center of the driven gear is held fixed and a torque is applied at the center of the driving gear, the teeth in contact and the bodies of both gears will deform. This condition yields an angular displacement of the center of the driving gear relative to the fixed frame of reference at the center of the driven gear. The relative angular displacement of the gears can be converted to a linear displacement along the line of action. The total relative displacement of the driving gear with respect to the driven gear along the line of action is defined as the static transmission error.

This paper discusses a computer simulation study in which the total amount of tooth profile modification and the length of the modification zone were systematically varied to determine their effect on the static transmission error and dynamic loading of spur gears. Both linear and parabolic modifications were studied. Their individual influence and relative significance on gear dynamic load are compared and discussed.

A gear set which operates at a constant design torque can be optimally modified to minimize its dynamic response. For gear systems that are to be operated at off-design torque or under variable loading conditions, design charts describing the gear dynamic load response due to different profile modifications are presented. The optimum length and amount of tooth profile modification may be determined from these design charts.

## THEORETICAL ANALYSIS

The theoretical model assumes that a simple spur gear transmission, which consists of a driving and a driven gears, two connecting shafts, a motor, and a load, can be treated as a lumped-mass vibration system (Fig. 1) (10,11). The motion of the system is expressed by the following set of differential equations.

$$J_M\ddot{\theta}_M + C_{s1}(\dot{\theta}_M - \dot{\theta}_1) + K_{s1}(\theta_M - \theta_1) = T_M$$

$$J_1\ddot{\theta}_1 + C_{s1}(\dot{\theta}_1 - \dot{\theta}_M) + K_{s1}(\theta_1 - \theta_M) + C_g(t)$$

$$[R_{b1}\dot{\theta}_1 - R_{b2}\dot{\theta}_2] + K_g(t)\,[R_{b1}(R_{b1}\theta_1 - R_{b2}\theta_2)] = T_{f1}(t)$$

$$J_2\ddot{\theta}_2 + C_{s2}(\dot{\theta}_2 - \dot{\theta}_L) + K_{s2}(\theta_2 - \theta_L) + C_g(t)$$

$$[R_{b2}\dot{\theta}_2 - R_{b1}\dot{\theta}_1] + K_g(t)\,[R_{b2}(R_{b2}\theta_2 - R_{b1}\theta_1)] = T_{f2}(t)$$

$$J_L\ddot{\theta}_L + C_{s2}(\dot{\theta}_L - \dot{\theta}_2) + K_{s2}(\theta_L - \theta_2) = -T_L$$

Where $\theta_M$, $\theta_1$, $\theta_2$, and $\theta_L$ represent the rotations of the motor, the gears, and the load; $J_M$, $J_1$, $J_2$, and $J_L$ represent the mass moments of inertia of the motor, the gears, and the load; $C_{s1}$, $C_{s2}$, and $C_g(t)$ are damping coefficient of the shafts and the gears; $K_{s1}$, $K_{s2}$, and $K_g(t)$ are stiffnesses of the shafts and the meshing teeth. $T_M$, $T_L$, $T_{f1}(t)$, and $T_{f2}(t)$ are motor and load torques and frictional torques on the gears; $R_{b1}$ and $R_{b2}$ are base circle radii of the gears; t is time; and the dots over symbols indicate time differentiation.

In developing the above equations several simplifying assumptions were employed. The dynamic process is defined in the rotating plane of the gear pair, and the contact between gear teeth is assumed to be along the theoretical line of action. Damping due to lubrication, etc. is expressed as a constant damping factor (ratio of the damping coefficient to the critical damping coefficient.) From gear research literature, typical damping factors of 0.10 and 0.005 respectively were chosen for the tooth mesh and and for the connecting shafts (12 to 14).

The stiffnesses and mass moments of inertia of the system components were found from the fundamental mechanics of materials principles. The equations of motion contain the excitation term due to periodic variation of the mesh stiffness and due to errors (such as spacing or profile errors). The meshing stiffness is a function of the mesh point along the line of action. Detailed analyses of the tooth meshing stiffness, shared tooth load, and static transmission error of the meshing gear pair were presented in previous studies (9,10).

Figure 2 presents a flowchart of the generalized computational procedure for the solution of the governing differential equations. The equations were linearized by dividing the mesh period into small intervals. A constant input torque $T_M$ was assumed. The output torque $T_L$ was considered to fluctuate as a result of time-varying stiffness, friction, and damping in the mesh.

To start the solution iteration process, initial values of the angular displacements were obtained by preloading the input shaft with the nominal torque carried by the system. Initial values of the angular speed were taken from the nominal system operating speed.

The iterative procedure was as follows: the calculated values of the angular displacement and speed after one mesh period were compared with the assumed initial values. Unless the differences between them were smaller than a preset tolerance, the procedure was repeated using the average of the initial and calculated values as new initial conditions. More complete descriptions of this method may be found in Refs. 9 and 10 and similar work appears in Refs. 4, 6 and 7.

The analysis was applied to a sample set of gears as specified in Table I. These are identical low-contact-ratio spur gears with solid gear bodies. The number of teeth is 28 and the module is 3.18 mm. Face width is 25.4 mm with a design load of 350 000 N/m (2000 lb/in). The gear mesh theoretical contact ratio is 1.64. A typical gear tooth showing both the unmodified (true involute) and modified profiles is illustrated in Fig. 3(a). A sample profile modification chart is shown in Fig. 3(b). On the chart, a straight line represents a linear tooth profile modification and a parabolic line represents a parabolic modification.

In this study, the same amount and the same length of profile modifications were applied to the tooth tip of both pinion and gear. The minimum amount of conventional tip relief was chosen as a reference value to normalize the amount of profile modification. Hence, for the minimum amount of conventional tip relief, $\Delta = 1.00$. Welbourn stated that the minimum tip relief should be equal to twice the maximum spacing error plus the combined tooth deflection evaluated at the highest point of single tooth contact (HPSTC)

(15). The length of profile modification is designated $L_n$. The distance along the tooth profile from tooth tip to the HPSTC is defined to be of unit length. The values of $\Delta$ and $L_n$ can be varied arbitrarily to obtain any desired combinations

Figure 3(b) shows examples of linear and parabolic profile modifications. In both cases the amount of modification $\Delta = 1.00$, and the modification length $L_n = 1.00$. Although the length of modification is shown as a vertical distance parallel to the tooth axis in Fig. 3(a), it is actually defined in terms of the gear roll angle as specified in Fig. 3(b).

## RESULTS AND DISCUSSION

Figures 4 and 5 show the comparison of the static transmission errors and shared tooth loads for unmodified gears and those with linear and parabolic tooth profile modifications. The normalized modification length $L_n$ was set at 1.0, which means the tip relief extended from tooth tip to the HPSTC location. The modification amount varied from $\Delta = 0.50$ to $\Delta = 1.25$ at an increment of 0.25. When the amount of profile modification was less than or equal to the minimum conventional tip relief, $(\Delta \leq = 1.00)$, the length of single and double contact zones as shown on the static transmission error graphs were not changed and the contact ratio remained at 1.64. When an excessive modification amount (for example, $\Delta = 1.25$) was applied on the tooth profile the zone of double tooth contact shortened and gear contact ratio was reduced (to approximately 1.53 for this case).

The principal excitation for gear system vibration is the unsteady component of the relative angular motion of meshing gears due to the variation of static transmission error. (The steady part of transmission error which is due to gear body "windup" does not cause excitation.) The main purpose of profile modification is to minimize this variation. A comparison of the conventional tip relief curves ($\Delta = 1.0$, $L_n = 1.0$) in the static transmission error plots of Figs. 4 and 5 shows that the linear profile modification curve is smoother than the one with parabolic modification. This indicates that if the conventional amount and length of tip relief is used, a spur gear system with linear profile modification is expected to provide a smoother dynamic response than gears with parabolic modification.

Figure 6 shows a speed sweep plot of the dynamic load factor for gears with no tip relief (unmodified), and gears with linear and parabolic tooth profile modifications (conventional modification amount and length: $\Delta = 1.0$, $L_n = 1.0$). The dynamic load factor is defined as the ratio of maximum dynamic tooth load during contact to static tooth load. The primary resonance for these cases occurs near fundamental system natural frequency 11,280 rpm. A Jacobi iteration technique was (16) used to determine the system natural frequencies. The peak value for the unmodified case was about 2.18. Peak values for the linear and parabolic cases were approximately 1.30, and 1.40 respectively. As above, the linear tip relief yields the smoothest response.

To understand the detailed effect of tooth profile modification on the dynamic behaviour of a spur gear transmission, the amount ($\Delta$) and length ($L_n$) were varied systematically. First, the effect of linear tooth profile modification on the dynamics of the sample gears was investigated. Figure 7 shows the speed sweep of dynamic load factor for the sample gear system with linear tooth profile modification running at design load. The normalized length was $L_n = 1.00$ and the amount was varied from $\Delta = 0.75$ to $\Delta = 1.25$. The dynamic response of a unmodified gear pair is also shown for comparison. As expected, the peak dynamic load factor at resonance speed is minimum at $\Delta = 1.0$ and $L_n = 1.0$. The maximum dynamic effect at $\Delta = 0.75$ was less than that at $\Delta = 1.25$. This result was anticipated in Fig. 4(a) where there is less variation in the static transmission error curve at $\Delta = 0.75$ than at $\Delta = 1.25$. This last result suggests that there is a greater detrimental effect of excess profile modification than of under modification. Excess profile modification reduces the contact ratio which increases the dynamic load.

Figure 8 shows the effect of varying load on the dynamic response of the sample gear set with conventional linear tip relief ($\Delta = 1.0$, $L_n = 1.0$). In Fig. 8(a), the applied load was normalized using the design load (350 000 N/m) as the reference value. (Ie: $W_n = 1.00$ when the appled load equals the design load.) At design load ($W_n = 1.00$), the value of the peak dynamic factor is 1.30. This is the minimum dynamic factor found. As the applied load varies from the design load, the maximum dynamic load factor increases from this value. From Fig. 8, load factor curves are shown at normalized applied load values ($W_n$) of 0.6, 0.8, 1.0, and 1.2. The corresponding peak values of dynamic load factor are approximately 2.47, 1.82, 1.30, and 1.45. These curves also show that underload ($W_n$ <1.0) produces a greater dynamic load factor than overload ($W_n$ >1.0). Finally, the dynamic load factor curve of an unmodified (involute) gear pair under design load is shown for comparison. The peak dynamic load value for unmodified involute gears is 2.18.

To obtain a more realistic feeling of the actual dynamic loading on the gear tooth and to prevent misleading interpretation, the speed sweep curves of Fig. 8(a) were replotted to show the actual tooth load in Fig. 8(b). The smallest peak value of the dynamic tooth load occurs for the design load case ($W_n = 1.0$). Both underload and overload cases show higher values of peak load. From the curves, maximum dynamic load values found are 518,700 N/m, 510,000 N/m, 455,000 N/m, and 609,000 N/m for $W_n = 0.60$, 0.80, 1.00, and 1.20 respectively. The detrimental effect of operating gears at a load substantially lower than the design load and at the resonant speed was clearly demonstrated. For at $W_n = 0.60$, the peak dynamic tooth load was actually greater than that at $W_n = 0.80$ and $W_n = 1.00$. Once again, the curve for unmodified involute gears running at $W_n = 1.0$ is shown for comparison. The benefit of gear tooth profile modification can be seen by comparing the dynamic tooth loads of modified and unmodified gears.

Similar studies were performed on the sample gears with parabolic tooth profile modifications. The results are presented in Figs. 9 and 10. Unlike the linear modification case, the minimum dynamic response of gears with parabolic profile modifications with $L_n = 1.00$ occured at $\Delta = 1.25$ instead of at $\Delta = 1.0$. This can be explained by comparing the static transmission error curves of these two cases in Fig. 5(a). At $L_n = 1.00$, the error curve for $\Delta = 1.25$ is smoother than that for $\Delta = 1.0$. This means gears with parabolic tooth profile modifications require a greater amount of modification than gears with linear profile modifications.

Figure 10 shows the dynamic response curves of gear pairs modified with parabolic tip relief at $\Delta = 1.00$ and $L_n = 1.0$ for various applied loads. The curve at $W_n = 0.8$ had the lowest peak value. Contrary to the linear case, gears with parabolic modifications run more smoothly at underload than at design load.

From the above observation, one may conclude that for conventional amount and length of profile modification ($\Delta$ = 1.00 and $L_n$ = 1.00), linear profile modification should be used for gears which will operate at and above design load, and parabolic profile modification should be applied to gears operating below design load, to minimize dynamic effect.

The various effects of applied load, profile modification length, and profile modification amount on the normalized maximum dynamic load of spur gears with either linear or parabolic tooth profile modifications were further investigated. The noramlized maximum dynamic load is defined as the product of the maximum dynamic load factor (MDLF) and the normalized total transmitted load ($W_n$). This normalized magnitude of the maximum dynamic load in the gear mesh provides better comparison of gear dynamics at different applied loads. Multiplying this normalized value by the design load gives the actual gear dynamic load.

First, a constant modification length of $L_n$ = 1.00 was assumed, and three different modification amounts of $\Delta$ = 0.75, 1.00, and 1.25 were applied to the sample gears. In Fig. 11 are plotted curves of the normalized maximum dynamic load over the load range of 0.70 to 1.20 times the design load ($W_n$). For the linear modification case, shown in Fig. 11(a), the normalized maximum dynamic load reaches a minimum value at 0.76 $W_n$ on the $\Delta$ = 0.75 curve and at 1.00 $W_n$ on the $\Delta$ = 1.00 curve. The minimum of the $\Delta$ = 1.25 curve apparently occurs at a load greater than 1.2 $W_n$ and is therefore off the scale of Fig. 11(a). The normalized maximum dynamic load appears to be more sensitive to load change for overload than for underload.

Figure 11(b) presents the dynamic load data for the parabolic modification case. On the curves for $\Delta$ = 0.75 and $\Delta$ = 1.00, the minimum dynamic effect occurs at a load less than $W_n$ = 0.70 and thus off the scale of Fig. 11(b). On the curve for $\Delta$ = 1.25, the minimum occurs at approximately 0.72 $W_n$.

Comparing the curves in Figs. 11(a) and (b) shows that the gears with parabolic tip relief are much less sensitive to changes in the amount of tip relief than gears with linear tip relief. Therefore, it is expected that the dynamics of parabolic tip relieved gears would be less affected by manufacturing tolerances and machining errors. In addition, the normalized maximum dynamic load for gears with parabolic relief appears to be generally lower than for gears with linear relief over the load range of $W_n$ = 0.7 to 1.2 (underload to overload). This means that parabolic tip relief is clearly a better choice than linear tip relief for gears that must operate over a wide range of loads.

The effect of different amounts of profile modification on the normalized maximum dynamic load of gears, at various applied loads in the range of $W_n$ = 0.7 to $W_n$ = 1.2, is shown in Fig. 12. As in the previous figure, the length of the modification zone was held constant at $L_n$ = 1.00. Figure 12(a) shows the curves for gears with linear modifications, and Fig. 12(b) for those with parabolic modifications. The optimum amount of profile modification for gears operating at either a single load or over a range of loads can be estimated from the minimum points on these curves. For the linear modification case, $\Delta$ = 1.00 is optimum for gears operating at the design load (constant $W_n$ = 1.0). If the gears operate over a range of loads, the optimum amount of modification is found from the intersection of the curves for the highest and lowest loads of the range. Therefore, for loads ranging from $W_n$ = 0.7 to $W_n$ = 1.0, the optimum modification occurs at $\Delta$ = 0.92 which corresponds to the intersection of the $W_n$ = 0.7 and $W_n$ = 1.0 curves in Fig. 12(a). Likewise, $\Delta$ = 1.18 is optimum for gears that operate from $W_n$ = 0.7 to $W_n$ = 1.2. For the parabolic modification case, it appears that $\Delta$ = 1.25 is the optimum amount for gears operating from $W_n$ = 0.7 to either $W_n$ = 1.0 or $W_n$ = 1.2. As noted above in the discussion for Fig. 11, the dynamic response of parabolically modified gears is less affected by the changes in the amount of profile modification than are gears with linear modification.

Finally, the effect of length of tooth profile modification on spur gear dynamic response was investigated and is shown in Fig. 13. The modification amount was held constant at $\Delta$ = 1.00. The length of modification zone was varied from $L_n$ = 0.50 to 1.30 and maximum dynamic load curves were generated for several values of applied load ($W_n$). The minimum dynamic response for gears with linear tooth profile modification occured at $L_n$ = 0.67, 0.78, 1.00 respectively for $W_n$ = 0.70, 0.80, and 1.00, see Fig. 13(a). Since gears seldom operate at a constant load in their daily operation a method must be found to choose profile modification specifications for the selected design load range. For the load range of 0.70 to 1.00 of design load ($0.7 < W_n < 1.0$), an optimum length for linear tooth profile modification is $L_n$ = 0.90. This value is obtained from the intersection point of the $W_n$ = 0.70 and $W_n$ = 1.00 curves from the normalized maximum dynamic load curves in Fig. 13(a). Any modification length other than this would yield less desirable higher dynamic effect under this range of loads.

A similar study for parabolic tooth profile modification is shown in Fig. 13(b). The applied load was varied from 0.70 to 1.20 of design load. (This is a wider load range than used for the linear case above, since we have shown that gears with parabolic modifications are suitable for a wider load range.) An optimum length of modification for minimum dynamic response for gears operating over a range of loads may be determined from this figure. For example: At constant design load, ($W_n$ = 1.0), the optimum length of modification is Ln = 1.30. For overload ($W_n$ > 1.0), the curves suggest that the optimum length will be greater than 1.30 (thus extending beyond the pitch point). In this study, modifications extending beyond the pitch point were not considered. As another example, if the operating load range is $W_n$ = 0.70 to $W_n$ = 1.00 (underload to design load), the optimum length is approximately $L_n$ = 1.28 (found at the intersection of the $W_n$ = 0.70 and $W_n$ = 1.00 curves). Finally, for a wider load range of $W_n$ = 0.70 to $W_n$ = 1.20 (underload to overload), the length of modification is chosen to be 1.30 (since this study does not consider modification extending beyond the pitch point). In general, a longer (than 1.0) length of modification zone seems to be preferred for parabolic tooth profile modification since it yields lower dynamic load.

A comparison of figures 12 and 13 shows that the modification length (Ln) has a greater impact on the maximum dynamic load factor than does the amount of modification ($\Delta$). Therefore the length of modification should be controlled as closely as possible. Nevertheless, due to machining errors and allowable tolerance it is not practical to manufacture tooth profile modifications exactly as specified by the theory. In reality, a modified tooth profile deviates somewhat from the ideal specification. As discussed earlier, parabolic profile modification appears to be less sensitive to manufacturing variance and is therefore preferred to linear profile modification.

As an example of designing the optimum parabolic tooth profile for a spur gear transmission operating at a range of loads, consider a gearset which operates over the load range between $W_n = 0.7$ and $W_n = 1.2$. Since the dynamic load is more sensitive to the length of modification ($L_n$) than to the amount ($\Delta$), $L_n$ is chosen first. From figure 13(b) the optimum length is 1.30. With the length $L_n$ fixed at this value, the optimum amount of profile modification can be found by varying $\Delta$ over a suitable range as shown in figure 14. This figure shows dynamic load curves at applied loads ($W_n$) of 0.7, 1.0, and 1.2 for gears with modification length $L_n = 1.30$, and modification amount varying from $\Delta = 0.75$ to $\Delta = 1.50$. The optimum amount of profile modification is found to be $\Delta = 1.18$. This is the intersection point of the $W_n = 0.7$ and the $W_n = 1.2$ curves. For this example, the worst case (highest value) of normalized maximum dynamic load will be 1.40. This is the load corresponding to the extremes of the range of applied load (at $W_n = 0.70$ and at $W_n = 1.20$).

## CONCLUSIONS

A computer simulation was conducted to investigate the effects of both linear and parabolic tooth profile modifications on the dynamic response of low-contact-ratio spur gears. The effects of the total amount of modification and the length of the modification zone were studied at various loads and speeds to find optimal (low dynamic response) specifications for profile modification. The following conclusions were obtained:

1. The amount and type of tooth profile modifications have a significant effect on the dynamic performance of spur gear systems.
2. Parabolic tooth profile modification is generally preferred for low dynamic response in gears which operate over a range of loading conditions. These gears are less sensitive to changes in applied load, amount of modification and length of modification than are gears with linear profile modifications.
3. Gears with parabolic profile modifications require a slightly longer length of modification zone than gears with linear profile modifications. The modification zone may extend beyond the highest point of single tooth contact.
4. Gears which operate at a nearly constant load at design load to moderate overload will perform more quietly (with less dynamic effect) if linear profile modification is used.
5. For gears with linear profile modification, excess modification has a greater detrimental effect on dynamic loads than under modification, and underload causes higher dynamic effect than overload.
6. Over a range considered in this report, the length of modification has a greater effect on the dynamic response for both linear and parabolic profile modifications than does the total amount of modification.

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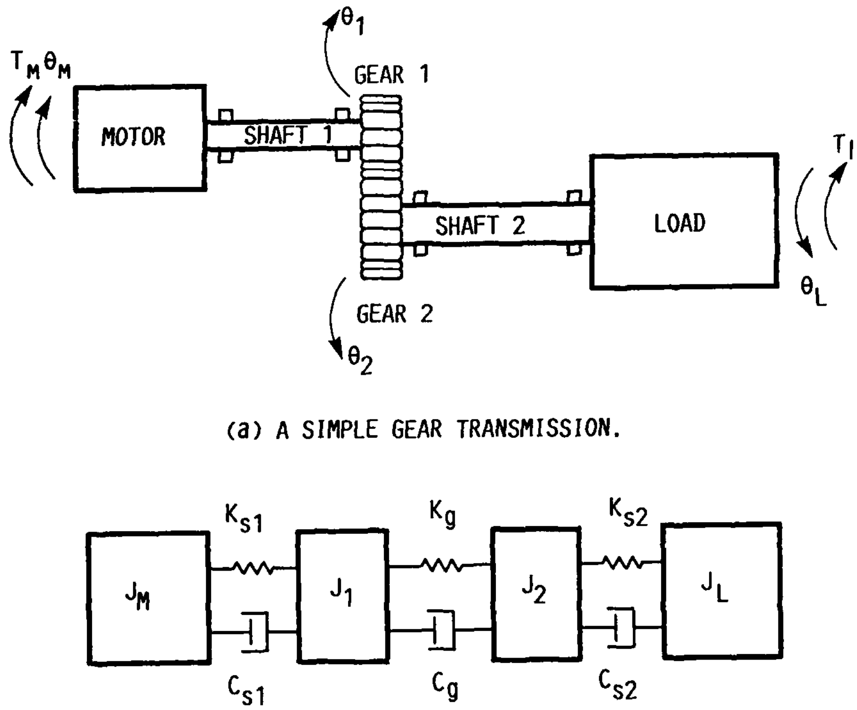
TABLE I. - GEAR DATA

| | |
|---|---|
| Gear tooth . . . | Standard involute full-depth tooth |
| Module, mm (diametrial pitch, in.$^{-1}$) . . . | 3.18 (8) |
| Pressure angle, deg . . . . . . . . . . . . . . | 20 |
| Number of teeth . . . . . . . . . . . . . . . . | 28 |
| Face width, mm (in.) . . . . . . . . . . . | 25.4 (1.0) |
| Design load, N/m (lb/in.) . . . . . | 350 000 (2000) |
| Theoretical contact ratio . . . . . . . . . . . | 1.64 |




(a) A SIMPLE GEAR TRANSMISSION.

(b) SYMBOLIC MODEL.

FIGURE 1. - COMPUTER MODEL OF SPUR GEAR SYSTEM.

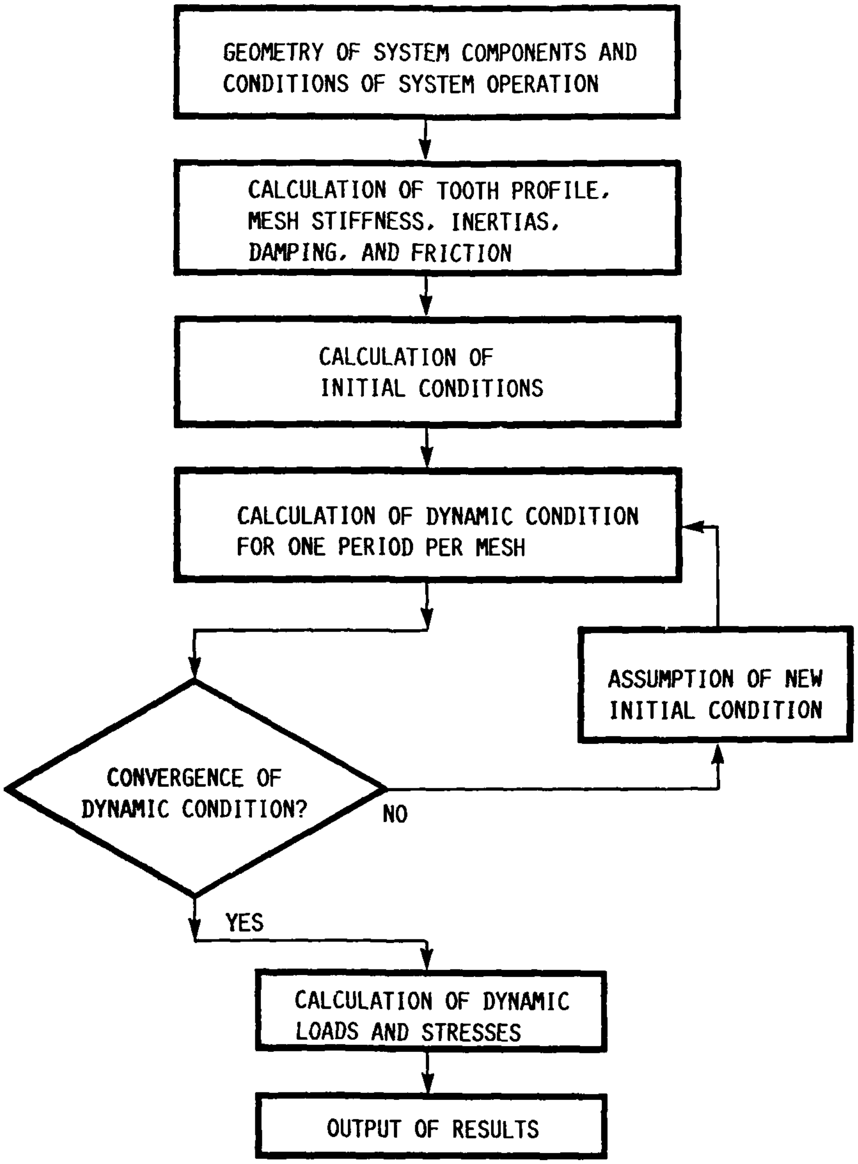
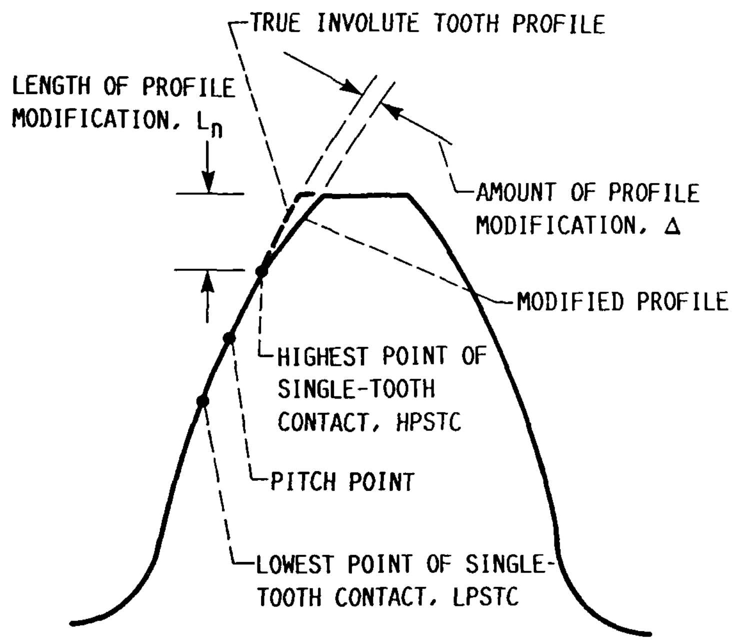
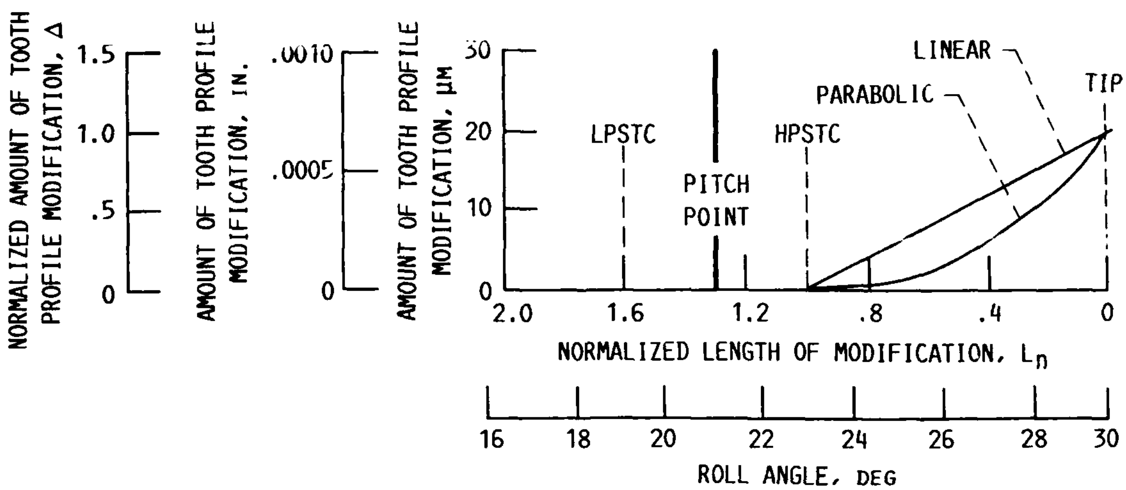



FIGURE 2. - FLOW CHART OF COMPUTATIONAL PROCEDURE.

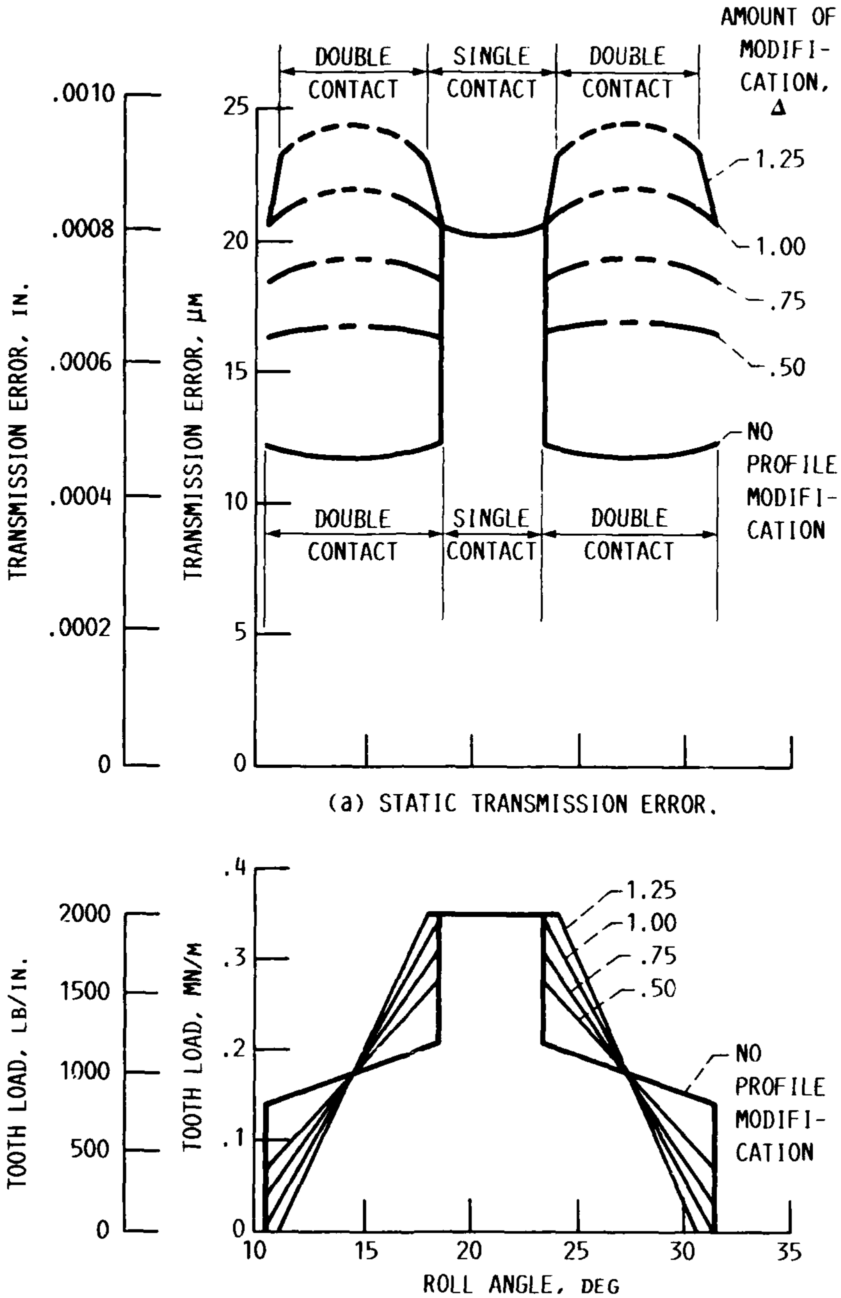



(a) GEAR TOOTH WITH MODIFIED TOOTH PROFILE.




(b) SAMPLE TOOTH PROFILE MODIFICATION CHART.

FIGURE 3. - EXAMPLE OF MODIFIED GEAR TOOTH.




(a) STATIC TRANSMISSION ERROR.



(b) SHARED TOOTH LOAD.

FIGURE 4. - STATIC TRANSMISSION ERROR AND SHARED TOOTH LOAD FOR GEAR PAIRS WITH LINEAR TOOTH PROFILE MODIFICATIONS. FULL DESIGN LOAD; LENGTH OF MODIFICATION, $L_n$ = 1.00.

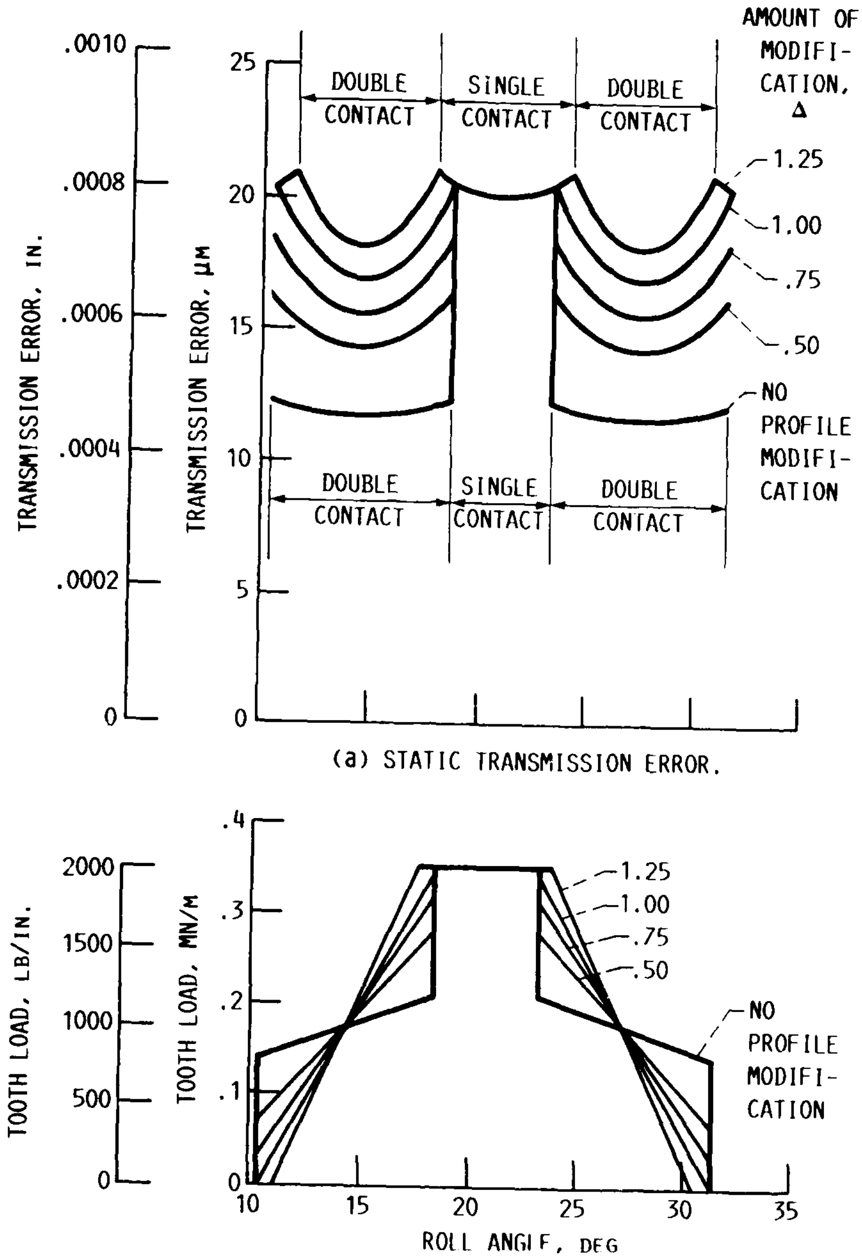



FIGURE 5. - STATIC TRANSMISSION ERROR AND SHARED TOOTH LOAD FOR GEAR PAIRS WITH PARABOLIC TOOTH PROFILE MODIFICATIONS. FULL DESIGN LOAD; LENGTH OF MODIFICATION, $L_n$ = 1.00.

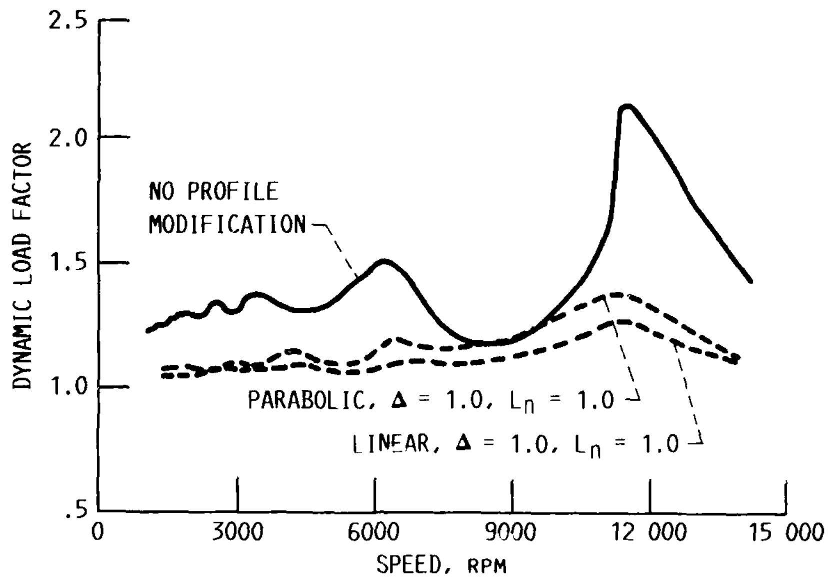



FIGURE 6. - DYNAMIC LOAD FACTOR OF SPUR GEAR PAIRS UNDER DESIGN LOAD WITH TRUE INVOLUTE TOOTH PROFILE, LINEAR PROFILE MODIFICATION, AND PARABOLIC PROFILE MODIFICATION.

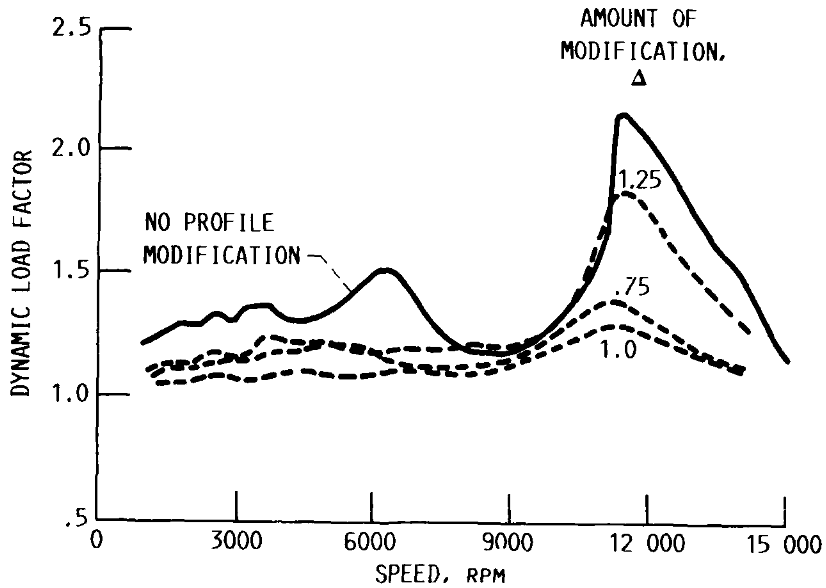



FIGURE 7. - EFFECT OF VARYING AMOUNT OF LINEAR TOOTH PROFILE MODIFICATION ON DYNAMIC LOAD FACTOR OF SPUR GEAR PAIR. FULL DESIGN LOAD; LENGTH OF MODIFICATION, $L_n$ = 1.00

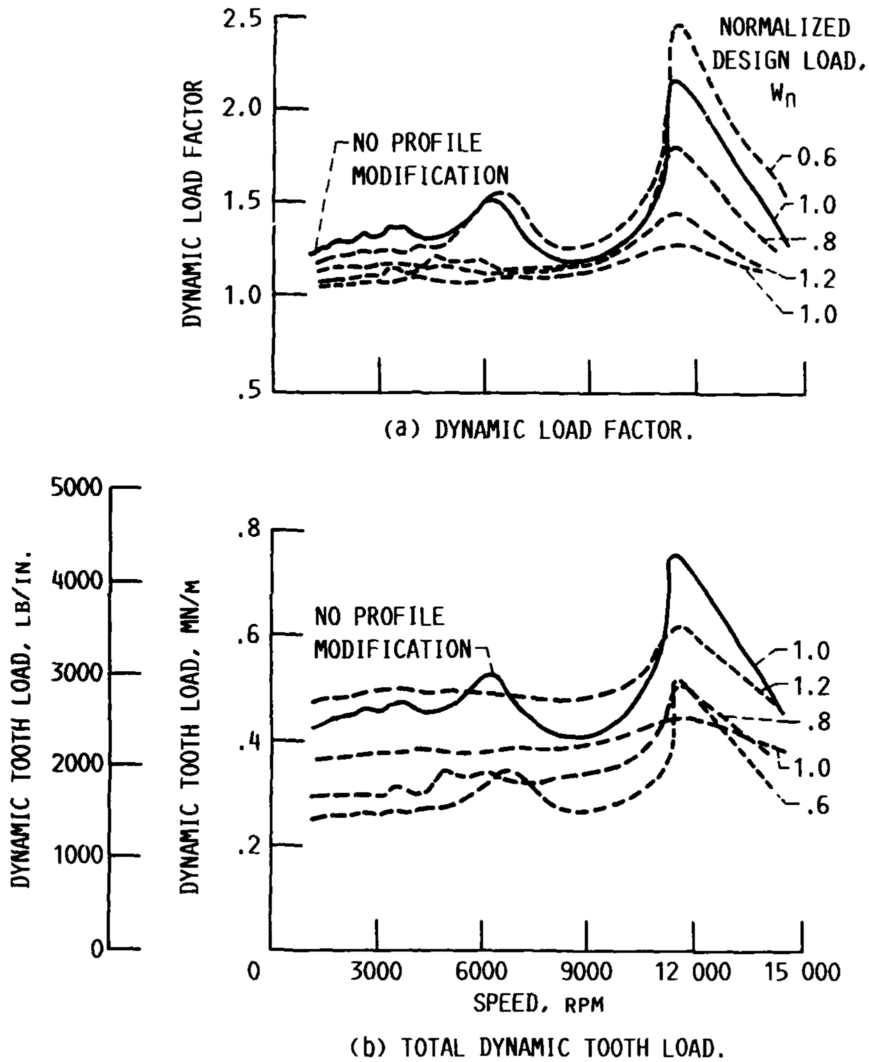



FIGURE 8. - EFFECT OF VARYING APPLIED LOAD ON DYNAMIC LOAD FACTOR AND TOTAL DYNAMIC LOAD OF SPUR GEAR PAIR. CONVENTIONAL LINEAR TIP RELIEF; LENGTH OF PROFILE MODIFICATION, $L_n$ = 1.0; AMOUNT OF PROFILE MODIFICATION, $\Delta$ = 1.0.

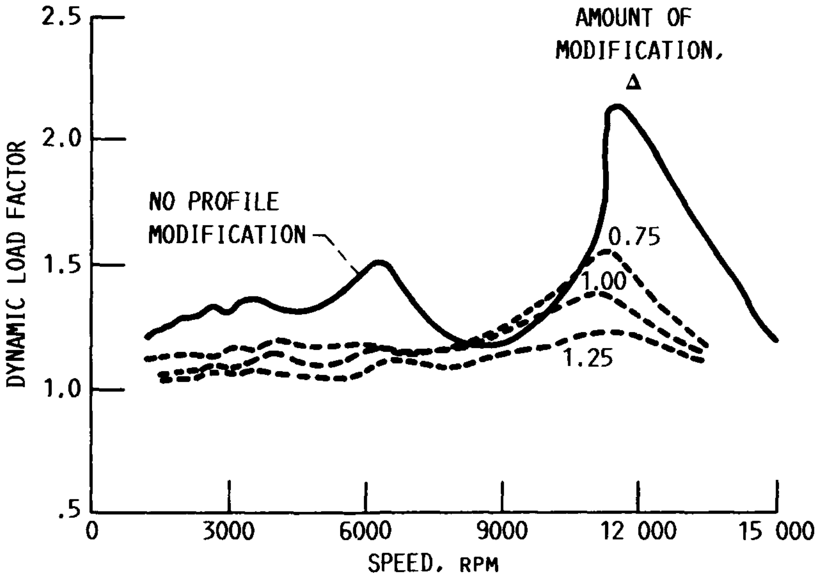



FIGURE 9. - EFFECT OF VARYING AMOUNT OF PARABOLIC TOOTH PROFILE MODIFICATION ON DYNAMIC LOAD FACTOR OF SPUR GEAR PAIR. FULL DESIGN LOAD; LENGTH OF MODIFICATION, $L_n$ = 1.00.

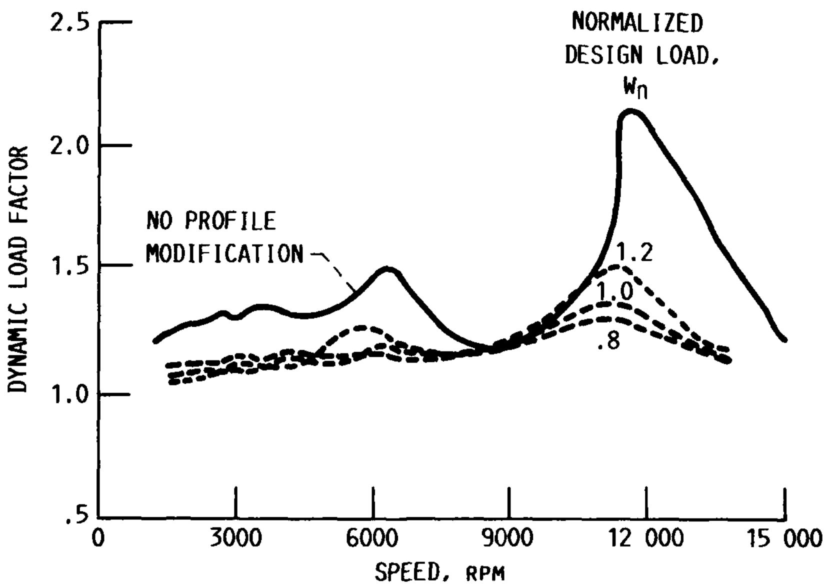
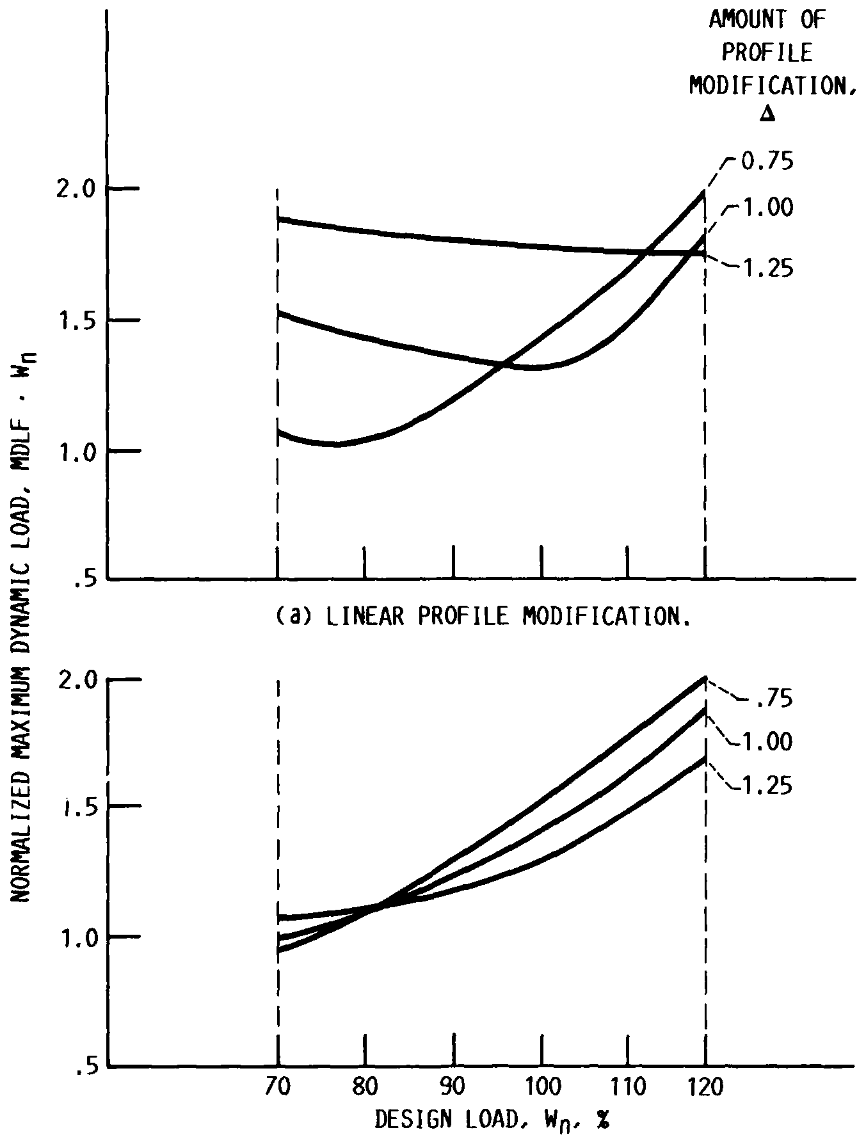
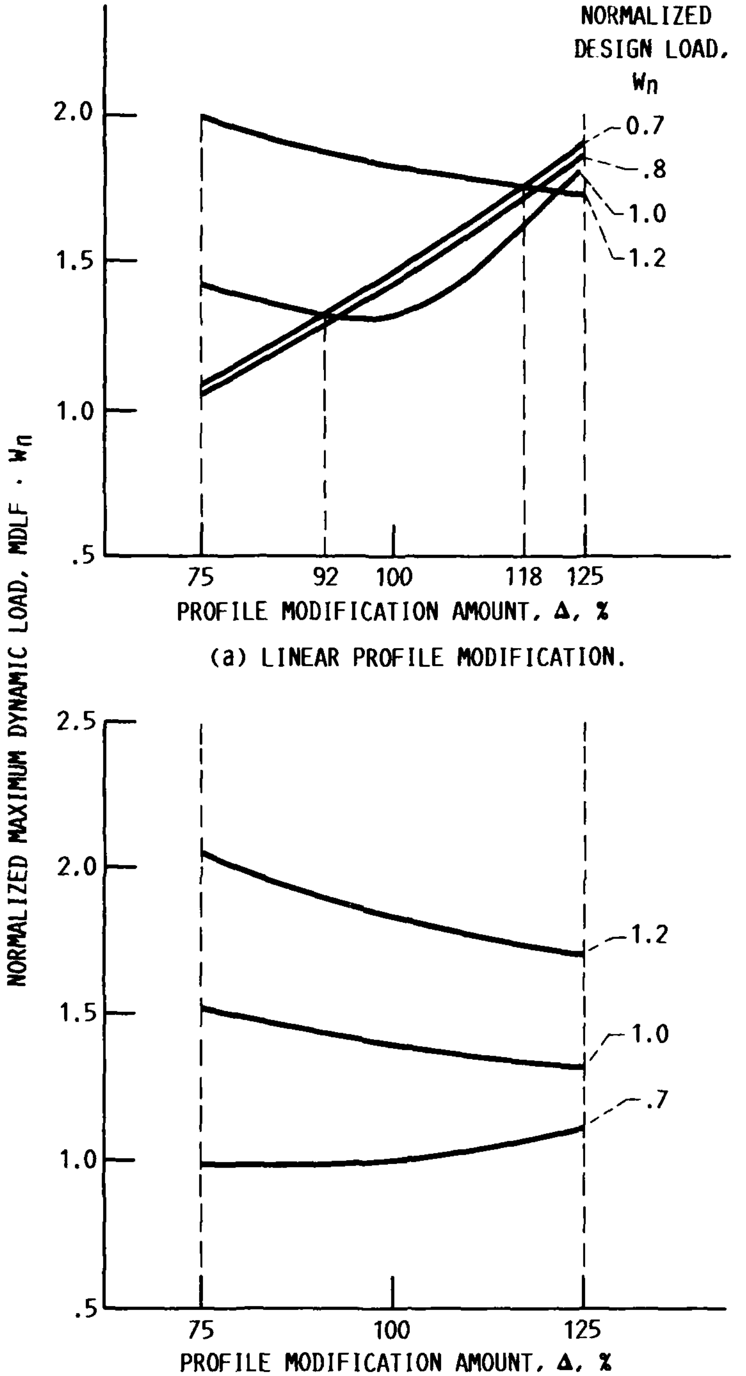



FIGURE 10. - EFFECT OF VARYING APPLIED LOAD ON DYNAMIC LOAD FACTOR OF A SPUR GEAR PAIR. PARABOLIC TIP RELIEF; LENGTH OF PROFILE MODIFICATION, $L_n$ = 1.0; AMOUNT OF PROFILE MODIFICATION, Δ = 1.0. (UNMODIFIED INVOLUTE CASE IS ALSO SHOWN FOR COMPARISON.)




(b) PARABOLIC PROFILE MODIFICATION.

FIGURE 11. - EFFECT OF APPLIED LOAD ON NORMALIZED MAXIMUM DYNAMIC LOAD OF SAMPLE GEARS AT VARIOUS MODIFICATION AMOUNT. LENGTH OF PROFILE MODIFICATION, $L_n$ = 1.00.




(a) LINEAR PROFILE MODIFICATION.

(b) PARABOLIC PROFILE MODIFICATION.

FIGURE 12. - EFFECT OF PROFILE MODIFICATION AMOUNT ON NORMALIZED MAXIMUM DYNAMIC LOAD OF SAMPLE GEARS AT VARIOUS APPLIED LOADS. LENGTH OF PROFILE MODIFICATION, $L_n$ = 1.00.

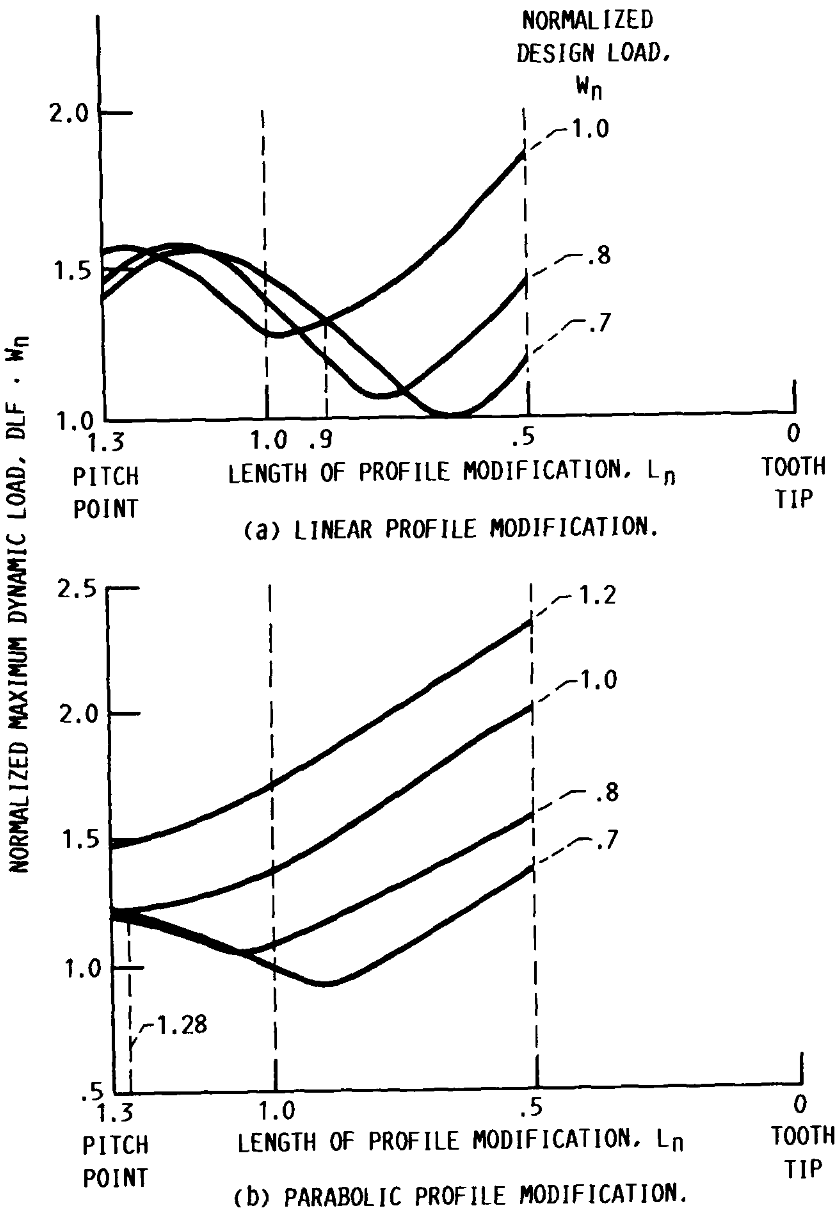



FIGURE 13. - EFFECT OF LENGTH OF PROFILE MODIFICATION ON NORMALIZED MAXIMUM DYNAMIC LOAD OF SAMPLE GEARS AT VARIOUS APPLIED LOADS. AMOUNT OF PROFILE MODIFICATION, $\Delta$ = 1.00

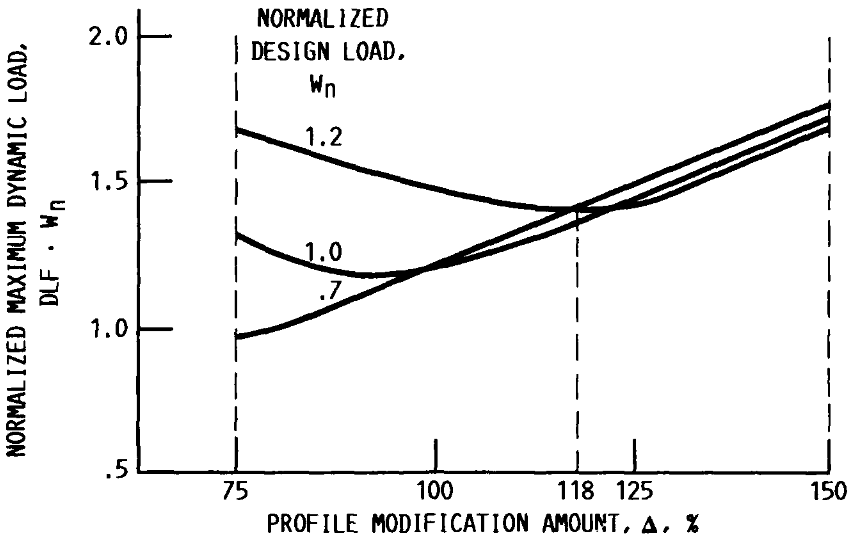



FIGURE 14. - OPTIMUM PARABOLIC PROFILE MODIFICATION FOR SAMPLE GEARS OVER RANGE OF APPLIED LOADS. LENGTH OF PROFILE MODIFICATION, $L_n$ = 1.30.

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**16. Abstract**

A computer simulation was conducted to investigate the effects of both linear and parabolic tooth profile modification on the dynamic response of low-contact-ratio spur gears. The effect of the total amount of modification and the length of the modification zone were studied at various loads and speeds to find the optimal profile modification for minimal dynamic loading. Design charts consisting of normalized maximum dynamic load curves were generated for gear systems operated at various loads and with different tooth profile modification. An optimum profile modification can be determined from these design charts to minimize the dynamic loads of spur gear systems. Keywords:

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